Coherent interaction effects in pulses propagating through a doped nonlinear dispersive medium

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Using a numerical approach we report on the cloning dynamics of simultaneous self-induced transparency (SIT) and nonlinear Schrödinger (NLS) solitons in a doped nonlinear dispersive medium. This technique involves a three-level atomic system interacting resonantly with two optical fields within a Λ scheme. As a result, a pulse in the signal frequency is transformed into a replica of the pulse in the pump frequency. The atomic population evolution shows that the basic mechanism behind the cloning process is the coherent population trapping effect. Furthermore, it is shown that the signal clone presents characteristics of both the SIT soliton and NLS soliton.

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I. INTRODUCTION

The investigation on coherent phenomena in connection with optical pulse propagation has been studied extensively by many authors. Particularly, the problem of propagation of a pair of matched optical pulses through a three-level medium under resonance conditions, has revealed effects that can be observed and further used for optical devices. For example, electromagnetically induced transparency [1], induced refractive index changes [2], and lately soliton cloning [3]. All of these effects have a common underlying mechanism, the coherent population trapping (CPT) effect, whereby an otherwise absorbing medium becomes transparent. The resonant interaction of the optical fields with the two electrical dipole transitions of a three-level system within a Λ scheme, provides a suitable situation to trap the atomic population in a coherent superposition of the lower states, so that the upper state remains practically unoccupied [4]. This kind of coherent interaction effects in atomic systems gives rise to many applications in high resolution spectroscopy [5], laser without inversion [6,7], atom-cooling systems [8], and pulse shaping [9]. Another example is given by electromagnetically induced transparency in nonlinear media, such as in rare-earth doped crystals [10], opening potential applications in high resolution image [11], and signal [12] processing. The application of coherent excitation to the investigation of nonlinear optical processes has also enhanced some known effects such as four wave mixing [13], second harmonic generation [14], sum-frequency generation [15], and giant nonlinearities [16]. In the propagation context, McCall and Hahn [17] have demonstrated the possibility of soliton pulse propagation through an absorbing medium modeled by a two-level atomic system, well known as the self-induced transparency soliton (SIT soliton). The SIT soliton appears as a stationary solution of the Maxwell-Bloch equations. Modeling the medium by a three-level system, Vemuri, Agarwal, and Vasada [3] demonstrated the phenomenon of SIT soliton cloning. The cloning process is understood as the amplification and shaping of a weak field of arbitrary temporal profile at the Stokes transition into a replica of a soliton wave at the pump transition.

Solitons are also present in dispersive nonlinear media [18] with important perspectives for fiber optical communication systems. This class of solitons represents solutions of the nonlinear Schrödinger equation (NLS) that governs nonlinear dispersive propagation away from resonances. By adding a dopant in the waveguide one may study coherent excitation effects, such as SIT, in the resonant propagation regime. In this case, the resonances modify the nonlinear polarization with an atomic contribution proportional to the coupling fields, but the soliton properties are still kept. Maĭmistov and Manykin [19] have reported the theoretical possibility on the coexistence of SIT and NLS solitons. Due to the development of doped optical fibers for optical communications, e.g., erbium-doped fiber amplifiers [20], this coexistence has been the subject of intense interest [21–24]. The numerical demonstration on the existence of this mixed state has been performed by Nakazawa, Yamada, and Kubota [21] who has named it as a SIT-NLS soliton.

Recently, the possibility of SIT-NLS soliton cloning has been demonstrated in a nonlinear dispersive medium coherently driven [25]. In this work, we extend the investigation on the propagation dynamics of a SIT-NLS cloned soliton, including results on the atomic population evolution and on the influence of a dispersive nonlinear medium over the cloning process as well as over the clone propagation. In Sec. II we present the basic theory and obtain the set of coupled equations that describe the propagation of the pulses (extended NLS equations) as well as the atomic system evolution (Bloch equations). Section III presents our numerical results based on numerical simulations of the set of coupled NLS-Bloch equations. Beyond the soliton cloning, the results for the atomic population behavior are also shown, illustrating the coherent population trapping effect that underlies the process of soliton clone formation. We further comment on the influence of cross-phase modulation (XPM) on this process through a phase variation analysis. Finally we demonstrate the possibility of controlling the temporal localization of the soliton clone under the variation of the ratio between the amplitudes of pump and signal pulses at the input, establishing an external control on the clone properties.

II. BASIC THEORY

Let us consider the dynamics of colored matched pulses propagating through a doped Kerr medium. By colored

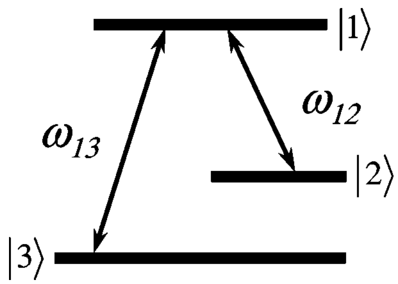

FIG. 1. Sketch of the energy levels for a Λ system.

matched pulses we mean a pair of optical pulses tuned in different frequencies (pump and signal) whose amplitudes and phases have a well-defined relation. We suppose that a three-level atomic system plays the role of a dopant in a nonlinear dispersive waveguide, e.g., an optical fiber. The system is then modeled by two resonant electromagnetic fields interacting with a three-level system within a Λ scheme including propagation effects such as self-phase modulation (SPM), cross-phase modulation and group velocity dispersion (GVD). The upper level, here characterized as the $|3\rangle$ state, is coupled to state $|1\rangle$ ($|2\rangle$) via a monochromatic field at frequency $\omega_1(\omega_2)$, as described in Fig. 1. Let $A_{13}(t)[A_{12}(t)]$ be the optical pulse tuned in a frequency close to the transition frequency ω_{13} (ω_{12}), denoted by pump (signal) pulse. Using the Schrödinger quantum description for this three-level system one may obtain the atomic evolution under the influence of both pump and signal fields. The Hamiltonian of the system can be written as $H=H_0+\mu E$, where H_0 is the free Hamiltonian, μ is the dipole moment of the system, and E represents the total optical field. Compared to the total Hamiltonian μE act as a perturbation. By first-order perturbation theory the system evolution is given by the following set of coupled differential equations for the probability amplitude $c_j(t)$ of the atomic levels $|j\rangle$ within the rotating wave approximation:

$$\frac{dc_1}{dt}=\frac{i}{\hbar}[c_2\mu_{12}A_{12}(t)+c_3\mu_{13}A_{13}(t)], \tag{1}$$

$$\frac{dc_2}{dt}=\frac{i}{\hbar}[c_1\mu_{12}A_{12}^*(t)], \tag{2}$$

$$\frac{dc_3}{dt}=\frac{i}{\hbar}[c_1\mu_{13}A_{13}^*(t)], \tag{3}$$

where μ_{12} and μ_{13} are the electrical dipole moments associated with the transitions. Together with the wave equation for the fields, this set of equations describes the intriguing effect of SIT, whereby a 2π-area pulse propagates without absorption. Particularly, if the pulse shape is a hyperbolic secant one has a SIT soliton. Starting from Maxwell's equations we can obtain the wave equation for the two optical fields. Here, the polarization describes all the effects involved: self-induced transparency, group velocity dispersion, and self-phase modulation due to the Kerr nonlinearity. Because of the presence of the two electric fields we need also to consider the effect of cross-phase modulation. Using the slowly varying envelope approximation and writing the electrical field as a z-propagating field, without considering the modal distribution, we obtain the following equations for the field envelopes:

$$\frac{\partial A_{12}}{\partial z}=-\frac{i}{2}\beta_{22}\frac{\partial^2 A_{12}}{\partial\tau^2}+i\gamma_2[|A_{12}|^2+2|A_{13}|^2]A_{12}+\frac{i\omega_{12}n_a}{2\varepsilon_0 c}c_2^*c_1\mu_{12}, \tag{4}$$

$$\frac{\partial A_{13}}{\partial z}=-\frac{i}{2}\beta_{23}\frac{\partial^2 A_{13}}{\partial\tau^2}+i\gamma_{13}[|A_{13}|^2+2|A_{12}|^2]A_{13}+\frac{i\omega_{13}n_a}{2\varepsilon_0 c}c_3^*c_1\mu_{13}, \tag{5}$$

where $\gamma_{1j}=n_2k_{1j}$ with n_2 as the Kerr coefficient related to the third-order susceptibility $\chi^{(3)}$ and k_{1j} as the wave vector corresponding to the field A_{1j}. n_a represents the dopant density and β_{2j} is GVD parameter associated to the field A_{1j}. It should be noted here, that Eqs. (4) and (5) are written in the reference frame that is moving with an average group velocity, that is, $\tau=(t-z/v_g)/T_0$ with T_0 as the pulse width and v_g as an average group velocity. In the presence of just one pulse (for example A_{13}) in a nonlinear dispersive medium, the nonlinear Schrödinger equation supports a soliton solution usually called the NLS soliton. The condition to obtain NLS solitons is that the sech shaped pulse power should be given by $|\beta_{23}|/\gamma_{13}T_0^2$. To investigate the soliton cloning in a nonlinear medium we use a coexistence soliton, a SIT-NLS soliton. The existence of such a soliton solution is subjected to the following condition [19]:

$$P_{0(\mathrm{NLS})}=P_{2\pi}, \tag{6}$$

which means that the power of a 2π SIT pulse must be such that the exact balance between dispersion and nonlinearity is attained. The set of equations (1)–(5) is analytically intractable and to obtain the evolution of both, pulses and atomic population, along the propagation one must employ a numerical approach. Using an algorithm that combines Runge-Kutta and split-step methods to simulate the coupled interaction, one is able to study the soliton cloning properties in the presence of group velocity dispersion, self-phase modulation, and cross-phase modulation.

All of the results described in the following sections are obtained by using the wavelengths for pump and signal as $\lambda_{13}=1.55\ \mu$m and $\lambda_{12}=1.45\ \mu$m and typical values for the fiber parameters such as the Kerr coefficient, $n_2=4\times10^{-16}$ cm^2/W. For simplicity, as the wavelengths are quite close we have used $\gamma_{12}\cong\gamma_{13}=n_2k_{13}$, and also $\beta_{22}\cong\beta_{23}=\beta_2=-2.207$ ps^2/km. Keeping in mind condition (6), we have used a peak power of 40 kW for the pump and 0.1 kW for the signal pulse and for both pulses we have used the same half width at half maximum of $\tau=1.763\times10^{-2}$ ps. Furthermore, as the energy differences between the resonant

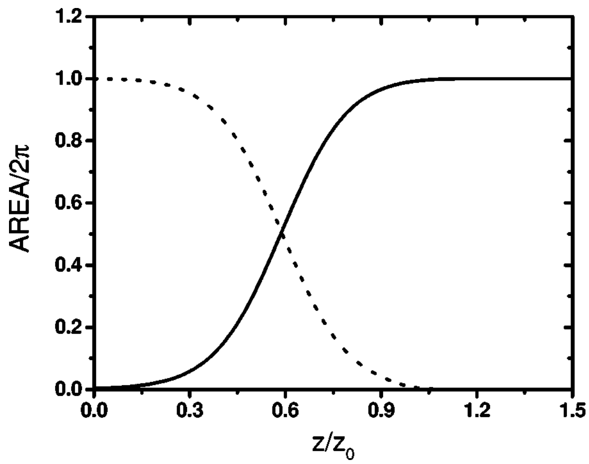

FIG. 2. Evolution of the areas of pump (broken line) and signal (full line) pulses as a function of the normalized propagation distance.

transitions are not large we have also used $\mu_{12}=\mu_{13}=\mu$. The dipole moment is then determined via condition (6) that gives $|\mu|^2=(n_2\omega\hbar^2/c\beta_2)$.

III. RESULTS

A. Soliton cloning

In the following we present our central result: the cloning of a SIT-NLS soliton. Let us begin by launching the two pulses pump and signal into the medium, in a situation where the pump pulse is a SIT-NLS soliton and the signal is 5% of the pump, and not necessarily a SIT-NLS soliton. The initial conditions for the set of differential equations are chosen so that the atomic population is in the lowest level that is, $c_3=1$, $c_1=0$, and $c_2=0$, and the input pulses are given by $A_{13}(\tau)=\sqrt{P_0}\,\mathrm{sech}\,\tau$ and $A_{12}(\tau)=0.05\sqrt{P_0}\,\mathrm{sech}\,\tau$. In Fig. 2 we illustrate the evolution of the areas of the pulses as a function of the normalized propagation distance z/z_0 to observe the characteristics of the SIT effect. Here z_0 is known as the soliton period defined by $z_0=\pi T_0^2/2\beta_2$. The pump pulse has an area equal to 2π and hence the quantum coherence along the propagation induces self-induced transparency. However, due to the presence of the signal, a process where energy from the pump is transferred to the signal pulse occurs. Within a distance of about $1.5z_0$ this process of energy transfer ceases and at this stage, the signal becomes a SIT-NLS soliton while the pump is depleted. To further illustrate the evolution of the pulses we plot both *pump* and *signal* for longer distances to show the soliton behavior of the *signal*. In Fig. 3 we note a signature of a NLS soliton since besides the fact that the shape of the signal does not change along the propagation, the characteristic delay from the SIT effect is not present. The pump evolution is described in Fig. 4 showing its depletion and illustrating the irreversibility of the energy transfer process. In this way, we can make a copy of a SIT-NLS soliton tuned at the pump frequency to a signal frequency through a mechanism of coherent interaction. An important observation of the soliton cloning process that occurs within a nonlinear dispersive medium is whether there are any frequency chirp along the clone after the process of energy transfer due to the presence of XPM. For the signal pulse the XPM effect can be seen through the frequency chirp along the pulse as shown in Fig. 5, represented by the dotted line. Notice that the XPM effect is already noticeable at a propagation distance of $z_0/100$ that is a small distance

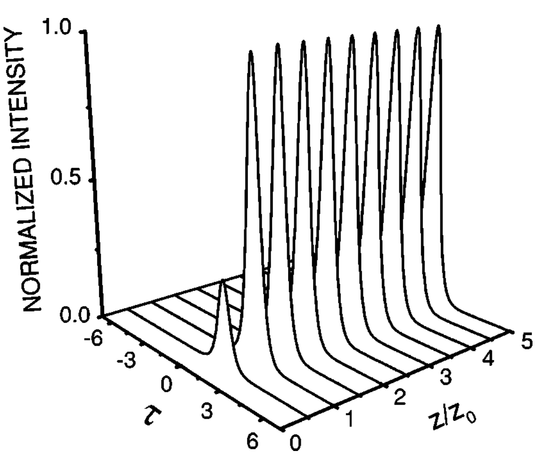

FIG. 3. Intensity profile of the signal pulse as a function of the normalized propagation distance z/z_0 and normalized time τ.

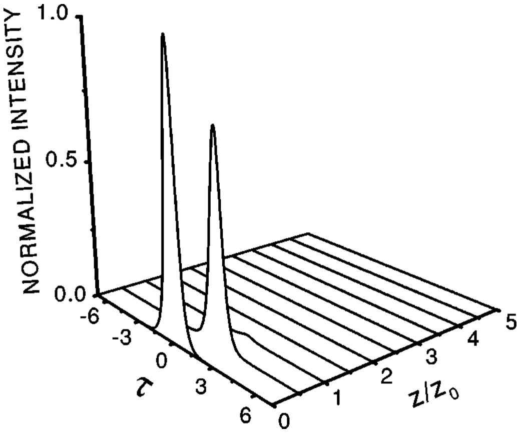

FIG. 4. Intensity profile of the pump pulse as a function of the normalized propagation distance z/z_0 and normalized time τ.

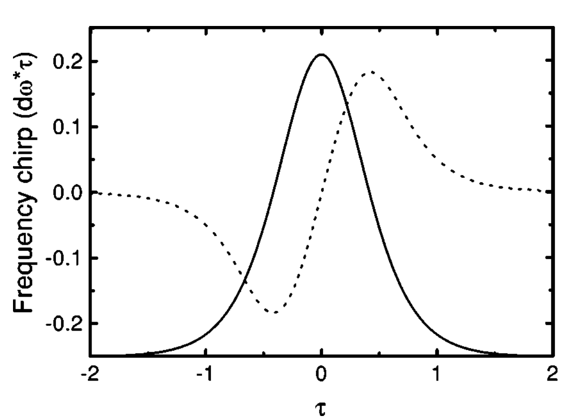

FIG. 5. Temporal profile of signal pulse (full line) superposed by its frequency chirp (dotted line) at a propagation distance of $z_0/100$.

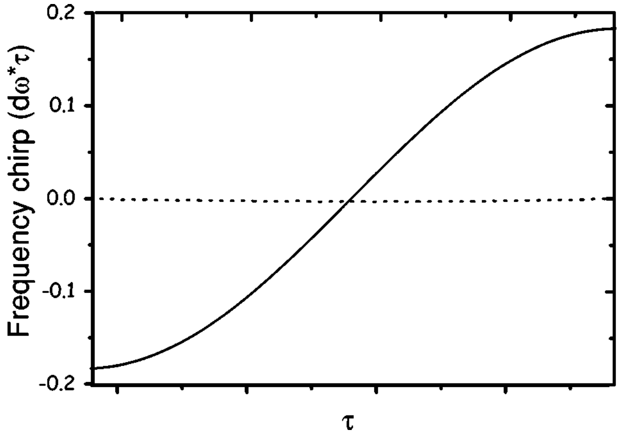

FIG. 6. Frequency chirp of the signal pulse around its central region at propagation distances $1.5z_0$ (dotted line) and $0.01z_0$ (full line).

compared with the propagation distance necessary for the energy transfer to occur. However, when the process energy transfer ceases the frequency chirp is zero characterizing a NLS soliton, since the solution of NLS equation does not present a frequency chirp. In fact, the vanishing of the pump ceases the XPM effect and together with CPT leads to zero frequency chirp transferring the NLS soliton characteristics to the signal pulse or one might say, making a copy of the pump at the signal frequency. Fig. 6 shows the frequency chirp along the pulse at a distance $1.5z_0$.

B. Coherent population trapping

Now we proceed to show the absence of the SIT delay and to confirm the SIT-NLS soliton behavior. While the signal propagates as a soliton pulse, a coherent population trapping effect is going on within the atomic system, which keeps all the atomic population in the ground state $|3\rangle$. This statement may be confirmed through a detailed study on the population evolution described by the Bloch equations. To this end let us turn to Figs. 7, 8, and 9 where the dynamics of the populations of states $|3\rangle$, $|2\rangle$, and $|1\rangle$, respectively, are depicted as functions of what we have named segment, which means the propagation distance inside the medium that corresponds to $v_g\Delta\tau$, where $\Delta\tau$ represents the pulse sample time. First, let us focus into Figs. 7 and 9. These figures show that in the first stage of the dynamics, a transition from level $|3\rangle$ to $|1\rangle$ appears in the central region of the pump since it is a 2π pulse. The probability to occur a transition between $|1\rangle$ and $|2\rangle$ is negligible because the signal area is not large enough to provoke a population inversion, as we can see in Fig. 8. Further ahead this situation starts to change and we note the energy transfer process illustrated above. This process amplifies the signal until it reaches an area large enough to promote a transition. At this stage the pump is not a 2π pulse anymore, as illustrated in Fig. 2. For a propagation distance around $0.7z_0$ a situation of coherent superposition between states $|3\rangle$ and $|2\rangle$ is established. Looking at the last stages of the propagation, we see that all population has gone to level $|3\rangle$ where it is trapped. Meanwhile, the area of the pump has faded away while the signal area

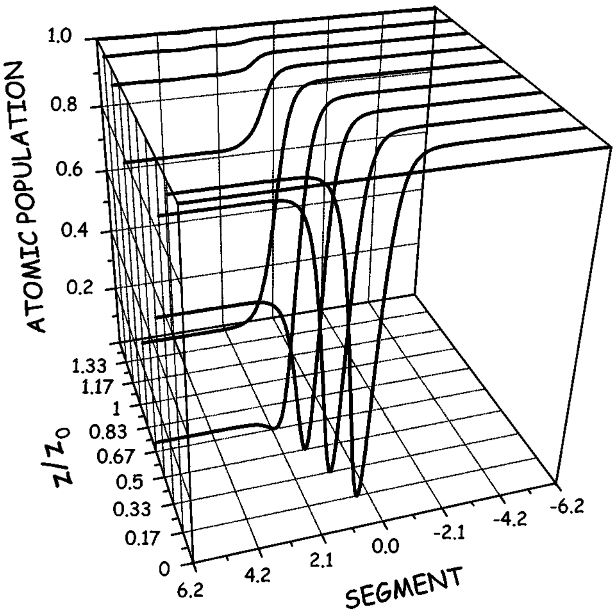

FIG. 7. Dynamics of the atomic population of the lowest level illustrating the coherent population trapping effect.

has become 2π. Since all the population is trapped at level $|3\rangle$ the signal does not feel the presence of atomic population and hence, it does not undergo absorption and re-emission such as in usual two-level SIT process. Therefore, once the energy transfer is completed, the three-level soliton does not present the characteristic SIT delay as it has been discussed in the literature [3]. The CPT effect explains the SIT soliton behavior presented by the cloned pulse, i.e., a self-preserving sech shape with an area equal to 2π.

C. Soliton dragging

Up to now all the results presented here for the soliton cloning were obtained by supposing that the initial signal

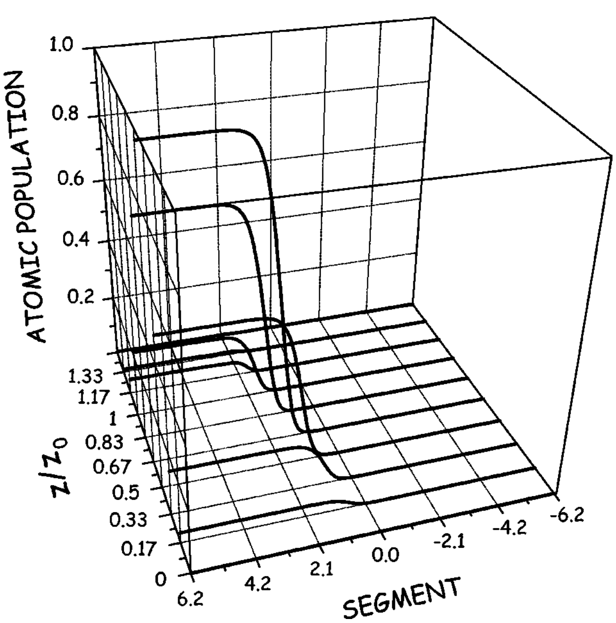

FIG. 8. Dynamics of the atomic population situated in level two.

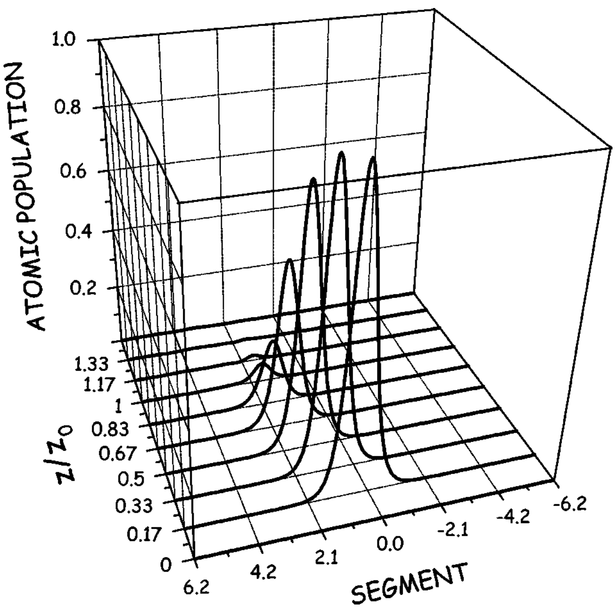

FIG. 9. Dynamics of the atomic population situated in level one.

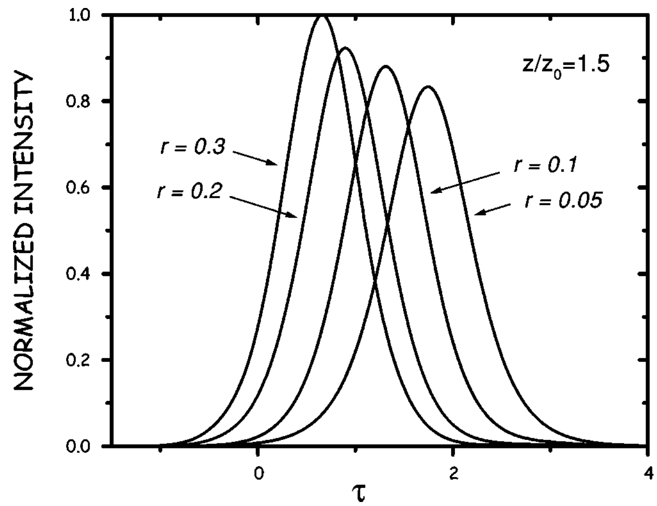

FIG. 10. Temporal profiles of signal pulse for various initial intensity ratios between pump and signal at a propagation distance of $1.5z_0$.

amplitude corresponded to 5% of the initial pump amplitude. In the following we shall obtain results for other values of the ratio of the amplitudes to show that, by varying this ratio one may control the final localization of the signal soliton a phenomenon first mentioned in Ref. [3] as soliton dragging. Observation of the soliton cloning effect have been verified for some values of the ratio $r=A_{12}(z=0,t=0)/A_{13}(z=0,t=0)$. Figure 10 shows the SIT-NLS soliton formation at $z/z_0=1.5$ for ratios of $r=0.05$, 0.1, 0.2, 0.3. We note that as the ratio grows the cloning process is sped up so that the clone generated under the larger rate is ahead of the clones generated within a smaller ratio. This may be understood by noting that as one grows the amplitudes ratio, the signal participates earlier on the whole process facilitating it. Studies on such aspects are in current progress and will be reported elsewhere.

IV. CONCLUSIONS

In conclusion, we have investigated through numerical simulations, the cloning of matched soliton pulses propagating in dispersive nonlinear waveguides including GVD, SPM, and XPM. Using a set of coupled NLS-Bloch equations to describe the coherent dynamics of a three-level dopant with resonances at the pump and signal fields, we have shown that the cloning mechanism is closely related to the population trapping effect. By varying the intensity ratio of the input pulses, we have demonstrated that the temporal location of the signal clone may be controlled. In the particular situation considered here we have demonstrated that the cloning process is irreversible. These results suggest that experiments carried out on doped fiber are possible, opening up many perspectives for optical applications. As an example, the effect of soliton cloning in doped fibers may be used to generate solitons streams at different wavelengths with potential applications in soliton multiplexing.

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